Path Diversification: A Multipath Resilience Mechanism

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***Abstract*—We present *Path Diversification*, a new mechanism that can be used to select multiple paths between a given ingress and egress node pair using a quantified diversity measure to achieve maximum flow reliability. The path diversification mechanism is targeted at the end-to-end layer, but can be applied at any level for which a path discovery service is available, e.g. intra-realm routing or inter-realm routing. Path diversification also takes into account higher level requirements for low-latency or maximal reliability in selecting appropriate paths. Using this mechanism will allow future internetworking architectures to exploit naturally rich physical topologies to a far greater extent than is possible with shortest-path routing or equal-cost load balancing. In this paper we describe the *path diversity* metric and its application at various aggregation levels. We then apply this metric to the path diversification process in the context of several real-world network graphs to asses the gain in flow reliability.**

I. INTRODUCTION AND MOTIVATION

Many of today's networked devices have access to multiple partial or complete physical layer paths between endpoints, but are unable to benefit from them due to design decisions in the current Internet protocol stack that assume unipath routing. Depending on the application, the benefits of using multiple paths can be in the form of improved performance, increased reliability, or both. In this paper we focus on the reliability aspects, recognizing that additional mechanisms are needed to optimize performance gains across multiple paths in real time (e.g. [1]).

The Internet protocol suite had survivability in the face of failures as a design goal [2]. It has also proven its resilience on a large scale, in large part due to the distributed nature of its components and operational protocols [3]. In spite of this, it quickly becomes apparent that there is a fragility to the performance of any given network application. Unseen perturbations in the network's operational state may result in an end-user experience which is far from optimal. Many applications attempt to disguise these lower-level failures, and some are quite successful, however this is only possible with significant programming overhead on a per-application basis. The goal of path diversification is to provide a unified interface to a service that is as reliable as the underlying physical graph, instead of being limited by the reliability of a particular path as in the current model. This could alleviate much of the programming overhead that currently exists at the application level, as well as providing a level of service not possible with the limited information currently provided to the end systems. At the same time, doing so requires that the end-to-end service be informed of application requirements in terms of throughput and upper delay bounds, given that there is an inherent tradeoff between maximizing path diversity and minimizing path stretch [4].

This paper presents a formal definition of the *Path Diversity* metric, and its aggregate properties when applied to node pairs and complete network graphs. Based on this notion of diversity we then present *Path Diversification*, which is the process of selecting the most advantageous subset of *available* paths to use for a given source-destination pair under particular application constraints. We then explore how path diversification improves reliability by comparing it both to the conventional unipath approach and the real connectivity of the underlying topology, which forms the upper performance bound in our case. We do not evaluate different path discovery mechanisms, but assume the availability of a path database that corresponds to the physical topology as has been proposed in the context of the PostModern Internetwork Architecture [5].

A. Terminology

While we try to maintain consistency with all widely accepted definitions, there are a few key terms defined here:

- Robustness: The ability of a system to maintain specified features when subject to assemblages of perturbations either internal or external. [6]
- Reliability: The ability to perform a required function under stated conditions for a specified period of time. [7]
- Resilience: The ability of the network to provide and maintain an acceptable level of service in the face of various faults and challenges to normal operation. [8] Resilience is a superset of many other metrics.
- Node pair: Any two nodes at the same hierarchical level of a particular network topology, e.g. two core nodes or two subscriber nodes.
- Path: Any complete set of nodes *and* links that form a loop-free connection between a node-pair.
- Path stretch: The ratio of the number of hops on a given path, divided by the number of hops on the shortest path.
- Flow: A data association between a node-pair which may be distributed over one or more paths.

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- Application: The higher-level entity that specifies the service requirements of a particular flow. This may refer to a traditional software application, or an alternative motivating factor, such as an SLA (service level agreement) in the context of an ISP network.

B. Design Goals

In order to achieve resilience, an end-to-end flow should be able to exploit diversity to the degree that it is present in the physical network graph. Current unipath mechanisms that rely on shortest-hop single-path routing cannot accomplish this. A resilient multipath mechanism should have the following design goals in providing service to higher layers:

- **High flow reliability:** Once established, a flow should remain stable as long as the underlying physical network is not partitioned.
- **End-system control:** The end systems or application should have some control over the paths selected.
- **Optimal paths:** The paths chosen should be the best available given the application's service requirements.
- **Minimal impact:** There should not be a negative impact on the network as a whole.

Section III-D formally describes the algorithm used in path diversification to meet these goals.

The remaining sections of the paper are organized as follows: In section II we present background and related research. Section III presents the path diversification mechanism itself. Section IV explains our evaluation methodology and presents our findings, and Section V concludes.

II. BACKGROUND AND RELATED WORK

While the current Internet architecture limits the use of multipath technologies, a significant amount of research has been done in this area. This section presents some characteristic examples of existing research and it's relation to path diversification.

A. Graph Theoretic Approach

The problem of finding paths through a network has been well studied in the context of graph theory as well as fiber network planning. The existing algorithms are based on different characteristics of these paths such as shortest paths, diverse, and disjoint paths [9]. Several algorithms exist to find the shortest path or k-shortest paths that include the earliest shortest path algorithms by Ford [10], Moore [11], Dijkstra [12], and Floyd [13], along with several modifications that address negative cycles and improve on or in some cases trade time and space complexities [14]. Following the shortest path between a pair of nodes, several algorithms were proposed to find the k-shortest paths, which involve simple techniques such as manipulation of edge weights to highly optimized algorithms [15].

Furthermore, the concept of diverse paths has been investigated to find a pair of diverse paths, k-diverse paths, and k-shortest diverse paths. The existing literature covers techniques based on shortest path algorithm with the incremental removal of used edges to graph transformations [16], [17]. Bhadari presents efficient algorithms to compute edge-disjoint and vertex-disjoint paths [14]. However, these algorithms are based on finding completely diverse paths. Bhandari also discusses an algorithm that finds the maximally diverse paths between a pair of nodes using a modified Dijkstra's algorithm.

In this paper, we use a method that draws heavily on Bhandari's maximally-diverse paths. However, we consider *both edge and node* disjoint diversity. We further expand the diversity measure by applying it to a set of nodes and to a full graph. We apply our algorithm to a network scenario in which the objective is to find k paths such that the diversity of individual node pairs as well as the overall diversity of the network exceeds a minimum threshold. Work has been done to characterize ISP networks in terms of the redundancy present in their physical layer graphs [18]. Our work is consistent with these efforts when applied in the same context.

B. Multipath Routing

Multipath at the network layer does not have the same end-to-end semantics as it does at the transport layer, however there have been a number of proposed multipath routing approaches that may have counterpart mechanisms at the transport layer.

Path Splicing [4] uses multiple destination-rooted routing trees to provide multiple alternative paths that may be switched between at any intermediate node. In this approach, the source node is given control over selecting a path index at each intermediate hop, however no information about the paths is passed to the source so there is no basis on which to chose a *particular* set of paths. The benefit is that if the source detects packet losses and suspects a bad link, it can randomly choose a different set of path indices much faster than routing can reconverge, however it has no assurance that the new path chosen will map to a different set of physical links. A similar approach is Routing Deflections in that the source node is given some control without detailed information [19].

An alternative, which is sometimes not even thought of as a multipath solution, is to have pre-computed back-up routes in case of a link failure [3]. While this can be faster than a full reconvergence, it still takes time for the nodes at the location of the failure to detect it and begin using the alternate routes.

Much of the multipath routing research, such as DETOUR [20], has been focused on the ad-hoc wireless environment in which channel conditions and resource constraints are the primary concern.

C. Multipath Applications

There are a number of proposed application scenarios (e.g. video streaming [21], [22]) that recognize the benefits of scheduling packets across multiple disjoint paths for performance gain or to mitigate the effects of bursty channel errors. Our goal with respect to these scenarios is to expose a well-defined set of control parameters (knobs) that allow the application to express its service level requirements without placing the full burden of discovering the optimal set of paths on the application itself.

III. Path Diversification Overview

The primary objective of path diversification is to select multiple paths that are as diverse as possible, while limiting the path stretch if necessary. Instantiating path diversification at any level will require the following four functions:

1) Path database indexed by source-destination pairs and including the unique identifiers for each node and link traversed. This database can be an exhaustive compilation or filtered based on administrative policy.
2) Quantified path diversity using the path diversity metric explained in the following section.
3) Path selection based on higher-level specifications to evaluate the tradeoff between path diversity and path stretch.
4) Packet forwarding based on the source routes distilled from the selected paths.

There are many possible implementations of the path database and packet forwarding mechanisms. We use them here to decouple the packet forwarding decision from the path discovery mechanism and thereby allow greater flexibility in exploiting the inherent diversity of heterogeneous internetworks [5]. The following sections discuss the measurement of path diversity and the selection of diverse paths.

A. Path Diversity

Since the primary motivation for implementing the path diversification mechanism is to increase resilience, paths should be chosen such that they will not experience correlated failures. To this end, we define a measure of diversity (originally introduced in [23]) that quantifies the degree to which alternate paths share the same nodes and links. Note that in the WAN context in which we are concerned with events and connections on a large geographic scale, a node may be thought of as representing an entire POP, and a link as the physical bundle of fibers buried in a given right-of-way. This distinction between WAN and LAN component identifiers affects only the population of the path database, not the usage of the diversity metric.

Definition 1 (Path): Given a (source s, destination d) node pair, a path P between them is a vector containing all links L and all intermediate nodes N traversed by that path

$$P = L \cup N \tag{1}$$

and the length of this path $|P|$ is the combined total number of elements in L and N.

Definition 2 (Path diversity): Let the shortest path between a given (s, d) pair be P_0. Then, for any other path P_k between the same source and destination, we define the diversity function $D(x)$ with respect to P_0 as:

$$D(P_k) = 1 - \frac{|P_k \cap P_0|}{|P_0|} \tag{2}$$

The path diversity has a value of 1 if P_k and P_0 are completely disjoint and a value of 0 if P_k and P_0 are identical. For two arbitrary paths P_a and P_b the path diversity is given as:

$$D(P_b, P_a) = 1 - \frac{|P_b \cap P_a|}{|P_a|} \tag{3}$$

where $|P_a| \leq |P_b|$.

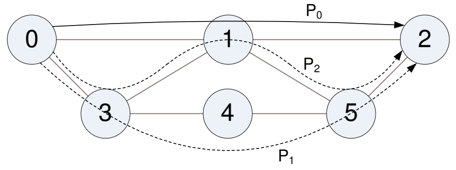

Fig. 1. Shortest path P_0 and alternatives P_1 and P_2

It has been claimed [4] that measuring diversity (referred to as novelty) with respect to *either* nodes *or* links is sufficient, however we assert that this is not the case. Figure 1 shows the shortest path, P_0, along with the alternate paths P_1 and P_2 both of which have a (link) novelty of 1. However, given a failure on node 1, both P_0 and P_2 will fail. In our approach, $D(P_2) = \frac{2}{3}$, which reflects this vulnerability. P_1 on the other hand has both a novelty of 1 and a diversity of 1, and does not share any common point of failure with P_0. Similarly, the wavelengths or fibers from multiple nodes may in fact be spliced into a single physical link such as was the case in the Baltimore Tunnel Fire [24], [25], resulting in a single point of failure, thus illustrating the need for including both nodes and links into the diversity measure.

B. Effective Path Diversity

Effective path diversity (EPD) is an aggregation of path diversities for a selected set of paths between a given node-pair (s, d). To calculate EPD we use the exponential function

$$\mathrm{EPD} = 1 - e^{-\lambda k_{sd}} \tag{4}$$

where k_{sd} is a measure of the added diversity defined as

$$k_{sd} = \sum_{i=1}^{k} D_{\min}(P_i) \tag{5}$$

where $D_{\min}(P_i)$ is the minimum diversity of path i when evaluated against all previously selected paths for that pair of nodes. λ is an experimentally determined constant that scales the impact of k_{sd} based on the utility of this added diversity. A high value of λ (> 1) indicates lower marginal utility for additional paths, while a low value of λ indicates a higher marginal utility for additional paths. Using EPD allows us both to bound the diversity measurement on the range [0,1) (an EPD of 1 would indicate an infinite diversity) and also reflect the decreasing marginal utility provided by additional paths in real networks. This property is based on the aggregate diversity of the paths connecting the two nodes.

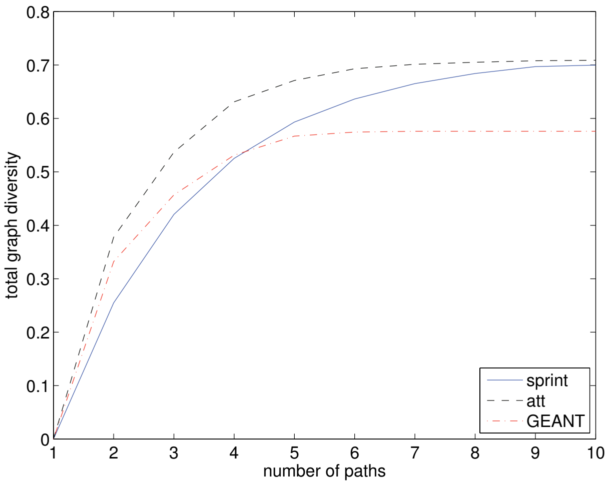

Fig. 2. Total graph diversity vs. number of paths selected

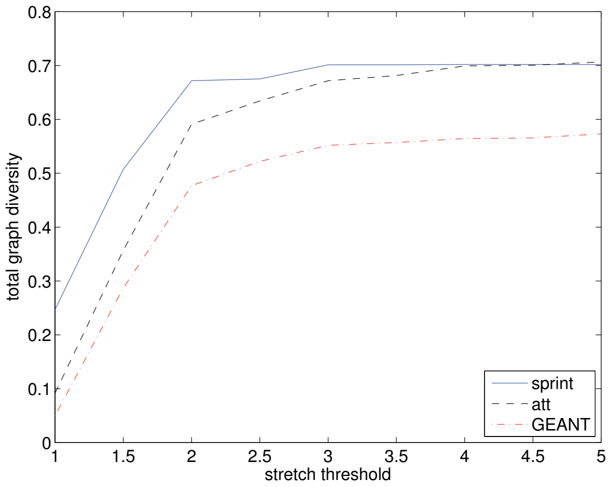

Fig. 4. Total graph diversity vs. path-stretch limit

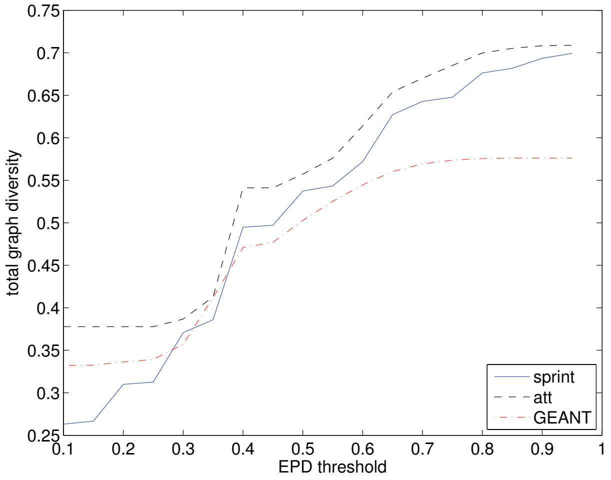

Fig. 3. Total graph diversity vs. effective path diversity threshold

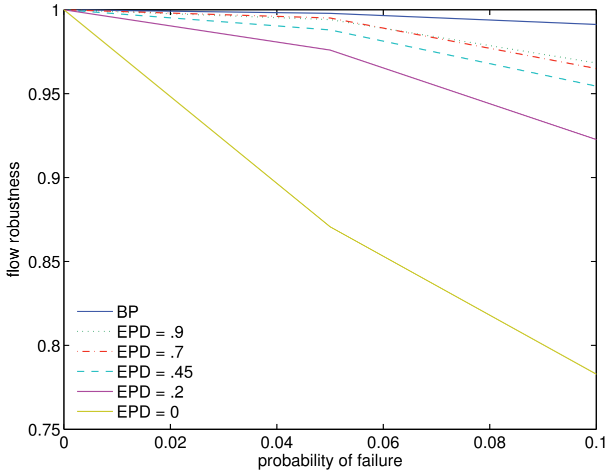

Fig. 5. Flow robustness vs. link failure probability for the AT&T topology

C. Measuring Graph Diversity

The total graph diversity (TGD) is simply the average of the EPD values of all node pairs within that graph. This allows us to quantify the diversity that can be achieved for a particular topology, not just for a particular flow. For example a star topology will always have a TGD of 0, while a ring topology will have a TGD of 0.6 given a λ of 1. In Figures 2, 3, and 4 we compare three different real network topologies' TGD plots. The significance of these results is discussed further in section IV-A.

Here we note that for diversity to make sense in the graph context it should be computed considering only path components (nodes and links) at the level of network hierarchy for which the diversity value is desired. For example, in computing the diversity of a service provider's backbone, only core nodes should be considered, otherwise the comparatively vast number of subscriber nodes (typically stubs) will artificially reduce the calculated diversity. We also note here that the diversity measure is designed such that it does not penalize longer paths in favor of shorter paths, meaning that graph diameter and average path lengths are independent metrics that should be considered in addition to the diversity metric.

D. Path Selection

In this section we use three different modes for choosing a set of diverse paths for a given node pair: number of paths, diversity threshold, and stretch limit. The objective in the first mode is to find k maximally diverse paths. We first find the shortest fully disjoint paths, and if additional paths are required we continue finding paths that add maximum diversity as calculated using equation 5. The second mode selects as many maximally diverse paths as are required to achieve the requested EPD. Finally, the third mode selects all maximally diverse paths with stretch less than the stretch limit. In all modes, the set of maximally diverse paths are found using the Floyd-Warshall algorithm with modified edge weights [14]. In this algorithm, only those paths are used that increase the EPD for the node pair in question. Recall that only paths with one or more disjoint elements (links,nodes) will result in non-zero $D_{\min}$ and consequently increase EPD.

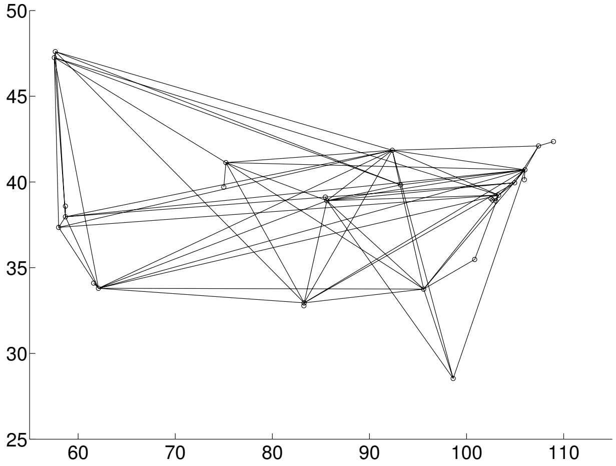

Fig. 6. Sprint topology

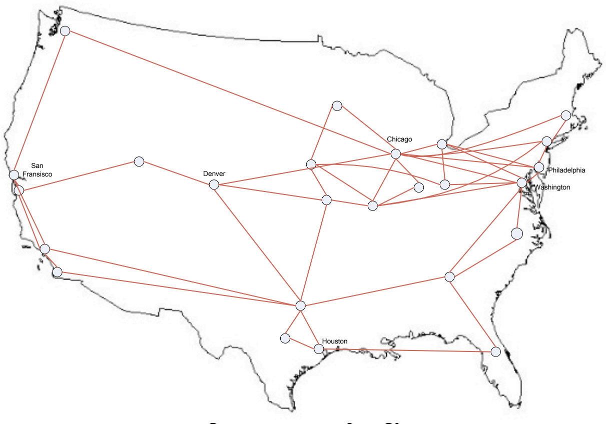

TABLE I
NETWORK STATISTICS

	Sprint	AT&T	GÉANT2
nodes	27	25	34
links	136	92	102

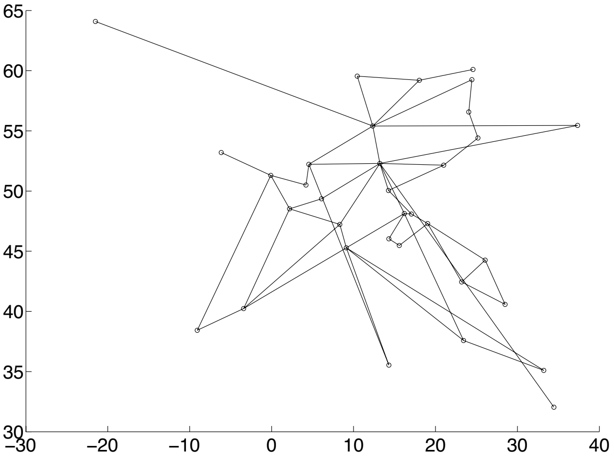

Fig. 8. GÉANT2 topology

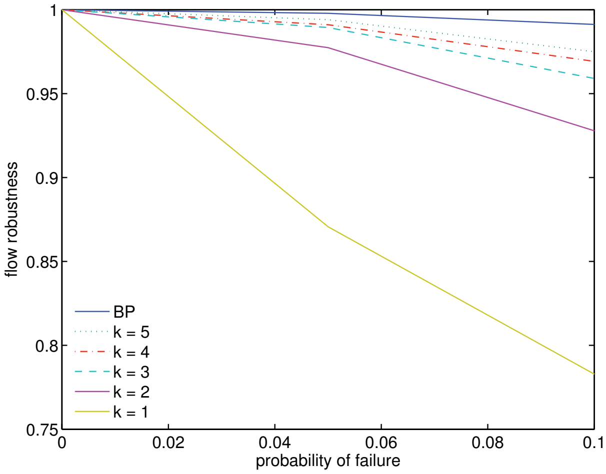

Fig. 9. Flow robustness vs. link failure probability for the AT&T topology

IV. EVALUATION

In this section we evaluate path diversification based on its ability to reflect the connectivity of the underlying graph, and the cost incurred in doing so in terms of path stretch. To evaluate path diversification based on realistic topologies we selected three service provider backbone network topologies, Sprint (US), AT&T (US) (from Rocketfuel database [26]), and GÉANT2 [27]. Since path selection mechanisms can only exploit diversity that exists at the physical layer, and not actually create diversity, we consciously selected well-connected topologies as shown in Figures 6, 7, and 8.

As will be seen in more detail in the results following, the characteristics of the topology significantly affect the diversity that can be attained. As mentioned earlier we assume the presence of a path server or equivalent service to provide the set of paths from which the selection is made. To perform this function we implemented a variation of the maximally diverse edge-disjoint path algorithm from [14]. We implemented a MATLAB simulation to evaluate the diversity measure and necessary supporting functions. Before examining the performance of path diversification with respect to flow reliability and stretch, we will look at its ability to improve the useable diversity of the graph as a whole.

A. Diversity

By applying the diversity measure to the selected path set of an entire graph, we observe the effectiveness of the path diversification process. To do this we select a set of paths

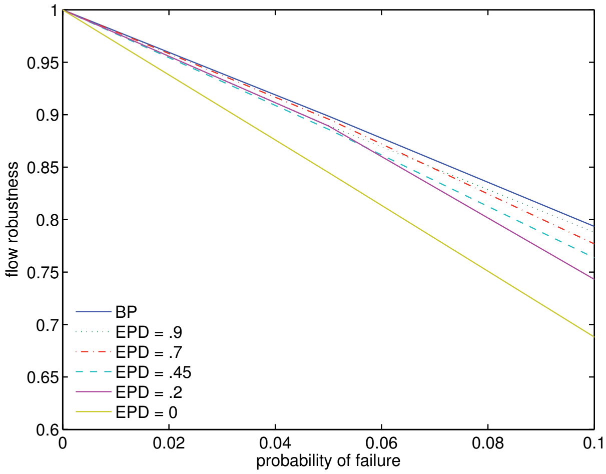

Fig. 10. Flow robustness vs. node failure probability for the AT&T topology

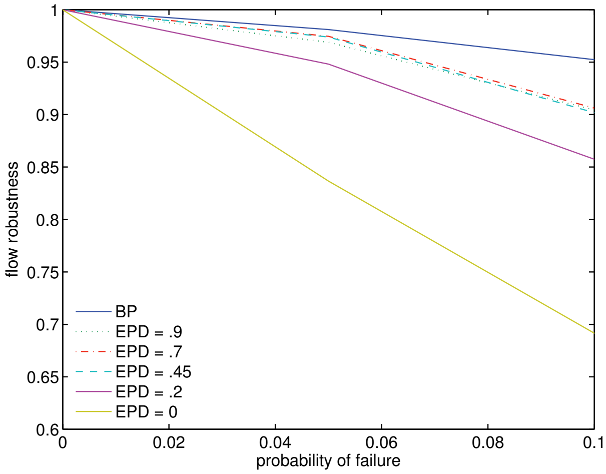

Fig. 11. Flow robustness vs. link failure probability for the GÉANT2 topology

based on a given k or EPD threshold, and then calculate the TGD based on that set of paths.

In Figures 3, 2, and 4 we observe that each of the three topologies share similar characteristics in terms of the diversity available, although the absolute value of diversity that can be attained varies. Selecting k most diverse paths between nodes in the AT&T and GÉANT2 graphs results in a strong increase in diversity for $k < 4$ with limited improvement for additional paths, as we observe in Figure 2, however the Sprint graph continues to show substantial improvement for $k < 6$. This emphasizes the fact that basing a diversity metric on a particular number of diverse paths is highly topology dependent. Figure 3 gives the TGD with respect to EPD, which shows similar relative diversity values of the three topologies, but aligns the knee of all three around an EPD value of 0.7, making this measure less topology dependent. From Figure 4 we observe that in all three cases, near-maximum diversity can be achieved while limiting the stretch of selected paths to 2.

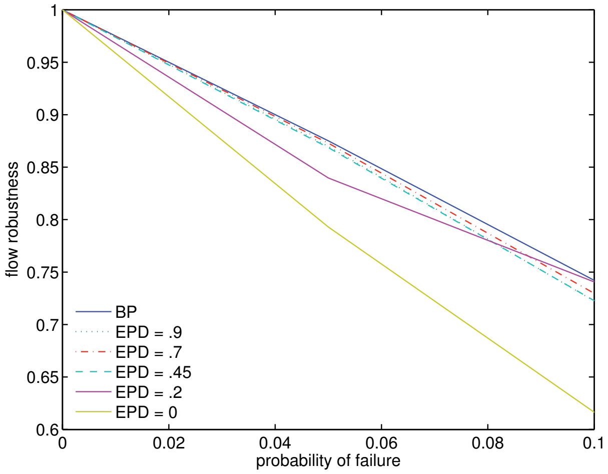

Fig. 12. Flow robustness vs. node failure probability for GÉANT2 topology

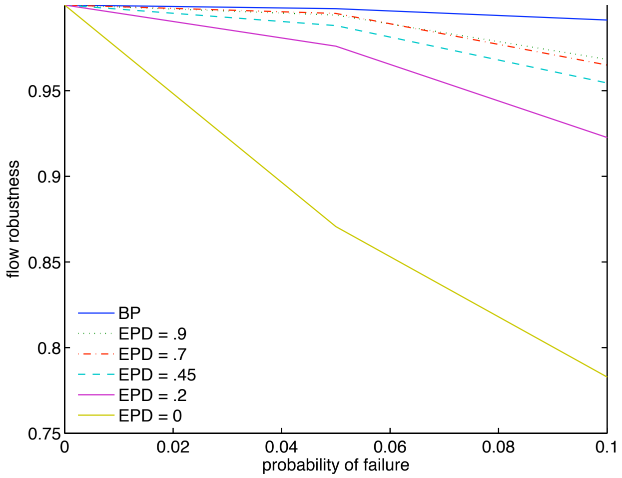

Fig. 13. Flow robustness vs. link failure probability for the Sprint topology

B. Reliability

A flow is established between each node pair using a set of paths determined using the path diversification algorithm and the specified diversity threshold value. To simulate link failures we remove each link from the graph based on a fixed probability of failure. To simulate node failures we remove all links connected to a particular node based on a fixed probability of failure for that node. A flow is considered *reliable* if at least one of its paths remains unbroken by the link or node failures. We compute *flow robustness* to be the number of reliable flows, divided by the total number of flows in the network. For each probability of failure, we also determine the the best flow robustness possible for any path-selection mechanism given the partitioning of the underlying graph, and show this value in each plot with the label BP. Finally, we, calculate the flow robustness using only the conventional shortest path for each node pair to serve as a lower performance bound.

To accomplish all this, we start with the set of paths between

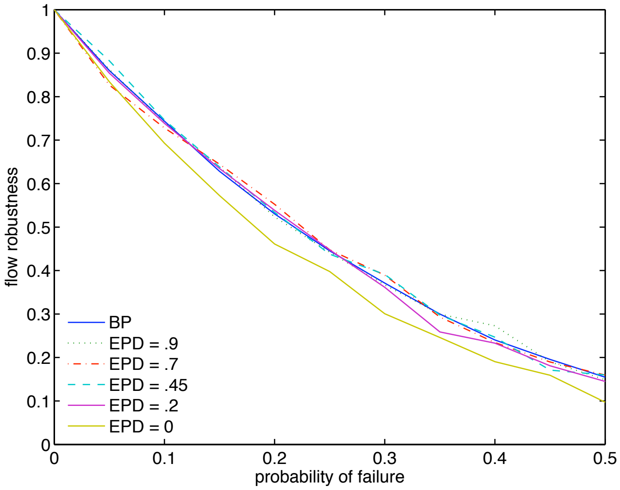

Fig. 14. Flow robustness vs. node failure probability for the Sprint topology

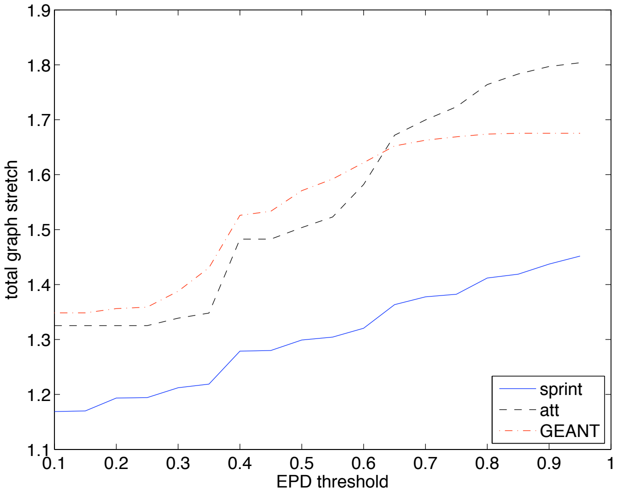

Fig. 15. Total graph stretch vs. effective path diversity threshold $\lambda = .5$

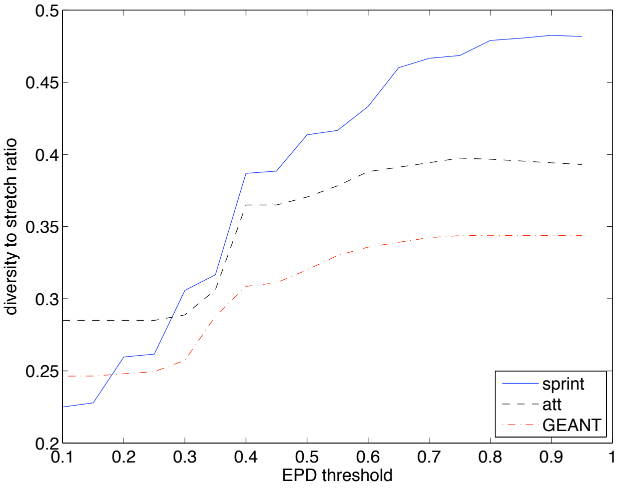

Fig. 16. Gain vs. effective path diversity threshold where $\lambda = .5$

all node pairs in the network selected by path diversification to meet a particular path-diversity threshold. We then remove each each edge of the graph independently with probability p. After calculating the resulting flow robustness we reset the graph and repeat the process 100 times for each probability p before continuing to the next path-diversity threshold. Each point plotted is the average of these 100 trials.

In Figures 5, 9, 11, and 13 we observe that seeking paths with even the lowest diversity threshold, be it k or EPD, provides a substantial improvement in robustness over conventional single-path selection when faced with random link failures. This correlates with the earlier plots of TGD that show the greatest improvements in graph diversity occurring at low values of k. In each of the plots we observe that near-optimal robustness is possible for link-failure probabilities below .1 (.25 in the case of the Sprint topology). Comparing Figures 5 and 9 shows equivalent curve sets selected using EPD ($\lambda = .5$) and k as the path selection criteria and shows that even relatively high diversity requirements (.9) can be met with relatively few paths. Figures 10, 12, and 14 apply the same measure to node failures instead of link failures. In these cases the best possible curve drops much more rapidly due to the inherent partitioning that occurs when a node fails (the flows between that node and all other nodes fail) in addition to the failure of any paths that transit through that node. In these cases we still show near-optimal performance with low EPD.

C. Path Stretch

By definition, if we are using paths other than the shortest-path as determined by conventional routing mechanisms, those paths will be longer, and this is measured in terms of *path stretch* as defined previously. Depending on the particular topology in use, and the source-destination pair being measured, some paths will be several times as long as the original, while others will be only slightly longer, and a particular flow will be composed of paths with varying stretch. To evaluate the effect of path diversification at the graph level we define *average path stretch* (APS) to be the average stretch over all selected paths between a give source-destination pair. The *total graph stretch* TGS is then the average of the APS values for all node pairs in the graph. We can then plot TGS with respect to EPD to determine the cost of increased diversity in terms of path stretch. As seen in Figure 15, the response of TGS to increasing EPD threshold is significantly topology dependent. The TGS of the Sprint topology increases at a relatively linear rate, indicative of the relatively high degree of connectivity in that network. In contrast the AT&T topology has distinct thresholds near EPD values of .4 and .6, due to the less homogeneous topology of that network. In all of the three networks we observe that the stretch remains less than double even for very high EPD thresholds.

While it is observed that stretch is likely to increase when selecting diverse paths, we want to observe the relationship between diversity and stretch. To this end we calculate a

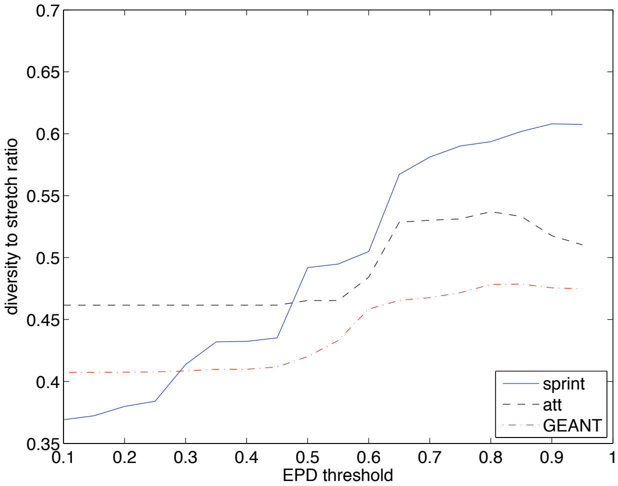

Fig. 17. Gain vs. effective path diversity threshold where $\lambda = 1$

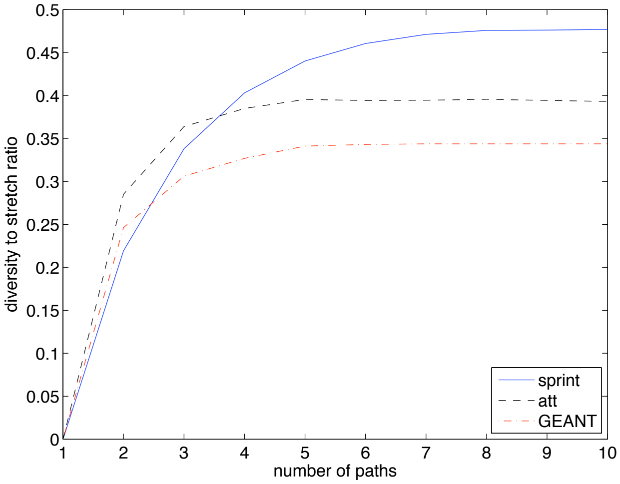

Fig. 18. Gain vs. number of paths requested

gain, which is the ratio of TGD divided by TGS for a particular requested EPD threshold. This normalizes the diversity improvement seen to the increased cost in terms of stretch. Figures 16, 17, and 18 quantify small increases in path diversity result in a large improvement in overall graph diversity, while incurring minimal stretch, however this effect is limited by the underlying topologies such that when no new diverse paths are available the ratio becomes constant. Comparing Figure 16 and Figure 17 we observe the effect of modifying λ. When λ is increased, the improvement in diversity does not come until higher EPD thresholds (due to the increased impact of k on EPD), however the overall gain is higher. Figure 18 shows the same gain measure when plotted against k, and again emphasizes the role played by the underlying topology. In this case the Sprint topology is still showing improved gain well after the other two have flatlined.

We also evaluate the relationship between flow robustness and stretch over a range of EPD thresholds for various failure probabilities. Figure 19 shows that EPD thresholds of up to 0.6 may be used before the combination of increasing stretch and declining marginal diversity cause the gain to drop noticeably. This behavior is most prominent when the probability of failure is very low, so the additional diversity is unnecessary but is nevertheless increasing the TGS of the network. Here we note that a multipath-aware application would likely not use all reliable paths equally, so averaging the stretch of all paths overestimates the effective stretch in that case.

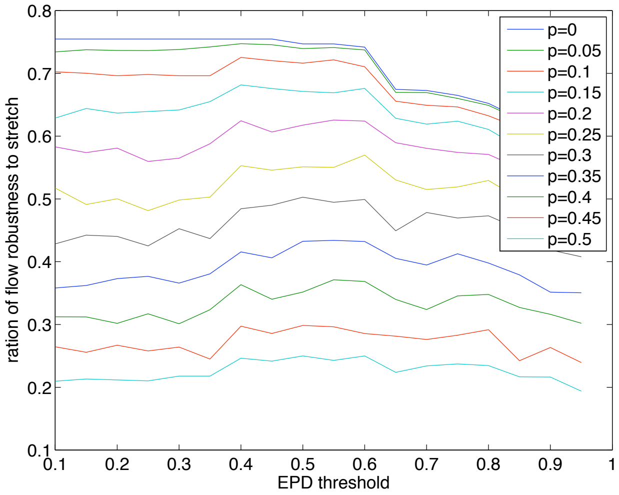

Fig. 19. AT&T gain vs. effective path diversity threshold $\lambda = 1$

V. CONCLUSION AND FUTURE WORK

This paper introduced *path diversification*, presenting its design and evaluation. We also discussed several metrics for evaluating path, node-pair, and graph diversity. We applied the path diversification mechanism to a number of real networks and evaluated its ability to improve flow robustness in the presence of link and node failures. Path diversification provides a substantial performance improvement over conventional single-path mechanisms by using cross-layer information to make intelligent path selections based on the diversity.

We select a static set of paths in order to limit complexity and bound the problem, however this is not strictly necessary. As paths fail additional paths could be requested from path server (assuming a resilient path to the path server). This sort of on-demand path selection could limit the initial computational complexity and is something that merits further investigation. Future work includes evaluating path diversification on a large set of generated topologies with specific properties (node degree, rank, etc.) to characterize the effect of those characteristics on graph diversity and flow robustness.

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